From Concrete to Abstract in the Measurement of Length

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Two Rasch Measurement Experiments that measure length

• Results in logits can be compared to deterministic measurement of length in mm
• The unit of length could be reproduced in logits
Experiment 1

- **Design**
  - Each of 99 matches broken in two at random
  - The length of 99 headed match sticks was compared with the length of 33 no-head sticks used as reference lengths

- **Scoring**
  - 1: Headed match longer than reference length
  - 0: Headed match shorter than reference length

- **Data**
  - 99 rows of 33 0s and 1s
  - 33 columns: each reference length vs 99 headed sticks

- **Joint Maximum Likelihood Estimation of length in logits**
The 99 headed matches
The 33 reference lengths
Experiment 2

• Design
  – 17 unevenly spaced units on ordinal ruler
  – At each hash mark: “Is the object at or above here?”
  – Range: 1.1 to 33.5 cm

• 42 physical objects of measurement
  – Various books, pictures, mirror, coffee mug, etc.
  – Range: 1 to 50 cm

• Scoring
  – 2: Object longer than reference length
  – 1: Object at reference length
  – 0: Object shorter than reference length

• Data
  – 42 rows: Each object by 17 reference lengths
  – 17 columns: Each reference length by 42 objects

• Modified pairwise comparison method
Guttman vs Rasch length probabilities
Same Design Elements Across Experiments

- Model (odds = B/D)

\[
Pr\{X_{ni} = 1\} = \frac{e^{\beta_n - \delta_i}}{1 + e^{\beta_n - \delta_i}}
\]

- Joint Maximum Likelihood Estimation
- WINSTEPS software (Linacre, 2013)
Experiment 1 Results

\[ \text{Length in mm} = 0.52 \text{ logits} + 21.7 \]
Experiment 2 Results

Logits and centimeters

Length in cm = 0.22 logits + 18
Discussion

• The two experiments reproduce length units from ordinal observations. They are analogous to other work making the same kind of point concerning weight, distance, and density.

Discussion

• “We do not in real life have perfectly spherical balls moving on perfectly smooth horizontal planes—the trick lay in the fact that it occurred to Galileo to imagine these."

• Galileo and others "were discussing not real bodies as we actually observe them in the real world but geometrical bodies moving in a world without resistance and without gravity..."

Discussion

• “A 'mathematical pendulum' is defined as 'a heavy point, swinging frictionless on a weightless string in vacuum'. A contraption like that was never seen; thus as a model for the motion of a real pendulum it is 'unrealistic'. Notwithstanding, it works quite well for a short time interval...”

Discussion

• “All models are wrong, but some are useful.”

• “Models should not be true, but it is important that they are applicable.”
Conclusions

• The demonstrated capacity to reproduce mm and cm lengths from ordinal observations suggests untapped potential for stable and metrologically traceable units of measurement in psychology and the social sciences.

• Success in devising and applying such units will require thorough qualitative elaboration of the constructs measured.